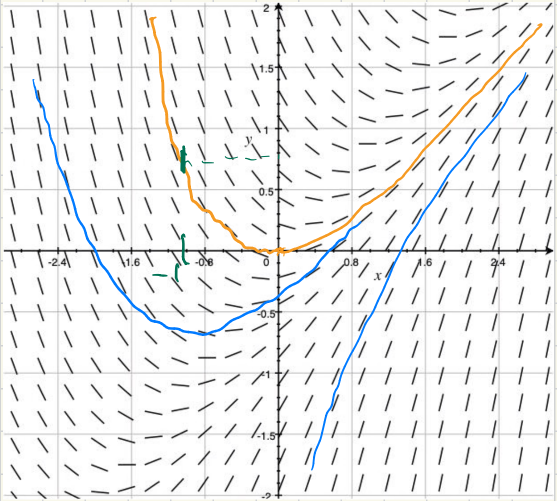
Section 1.3: Slope fields

We learn:

- what is a slope field?
- How to draw them and recognize them
- Features of a solution that we can identify
- A theorem about existence and uniqueness of solutions

Example (page 20): The slope field for dy/dx = x - y



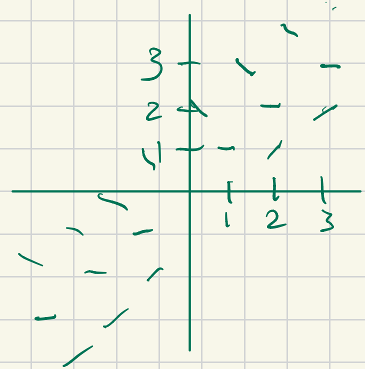
Follow along the lines to get a solution.

Like questions 21, 22: Construct this slope field.
Sketch the solution curve corresponding to the given initial condition.
Use this solution curve to estimate the desired value of y(x).

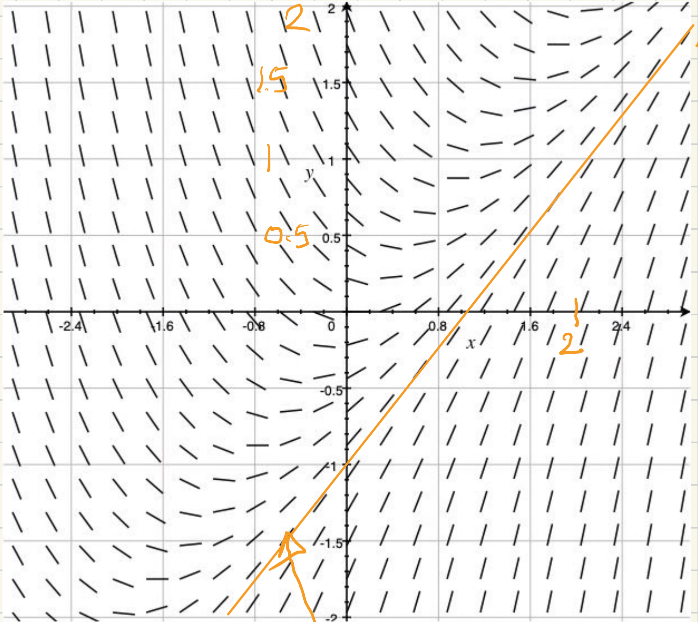
$y' = x-y, \ y(0) = 0, \ y(-1) = ?$ 0.8

The slope field for $\frac{dy}{dx} = f(x,y)$ has at each point (x,y) a line segment with slope $\frac{dy}{dx}$.

$y=x$ $\quad$ $y=x-1$



$y' = x-y$



We see:

- the solutions approach an asymptotic line
- Through each point there is a unique solution
- What else? Above the asymptotic line as $x < 0$, y gets large.

Question: If $y(0) = 0$, what is $y(2)$ closest to?

a. 0.5
b. 1
c. 1.5
d. 2

Theorem 1 (not tested)
Consider a differential equation $dy/dx = f(x,y)$.
If $f(x,y)$ and $\partial f/\partial y$ are continuous in some rectangle with (a,b) in the interior, then the d.e. has a unique solution with $f(a) = b$ on some open interval containing a.

Example $y' = x - y$. $x-y$ is continuous

$\frac{\partial(x-y)}{\partial y} = -1$ is continuous.

There is a unique solution for each initial condition.

Question: What is the difference between a slope field and a vector field?